Space Tug Performance Optimization

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Theme

IN keeping with the concept of reusability the space tug extends the realm of application to near Earth space. Missions which could benefit from the assistance of a tug are now being proposed.[1] From the performance analyst's point of view the trajectories required for these missions are somewhat more complex than those required for missions not employing a reusable assisting vehicle. In order to demonstrate some complexities and indicate one method for analyzing space tug trajectories a particular mission was chosen and is discussed here in some detail.

The basic mission profile to be considered is that of the tug-assist-to-escape maneuver. The trajectory is initiated from a circular base orbit. From the base orbit the tug accelerates a spacecraft to an intermediate orbit with energy somewhere between that of the base orbit and escape energy. After separation the spacecraft ignites its engine and accelerates to escape energy. The tug stays in the intermediate orbit for some time and then reignites its engine and returns to the base orbit. Similar results for nonthrusting spacecraft have been obtained by Weyer and Teren.[1]

Content

For reference the three burns are labeled in Fig. 1. Branch 1 represents the first tug burn, branch 2 the second tug burn and branch 3 the spacecraft burn. Each maneuver is two dimensional so the entire profile is planar. Maximization of the final spacecraft mass is used as a performance index. The intermediate orbit is left unconstrained; to avoid its degenerating to a parabola or hyperbola the total propellant consumed by the tug during its two maneuvers is held constant. Optimization must determine the steering profile and burn time for each of the three branches. In the process this will establish the optimal intermediate orbit. Other parameters such as the ignition point for the spacecraft and reignition point for the tug must also be determined.

This discussion describes a branched trajectory. Necessary conditions for optimal branched trajectories have been developed and applied to a variety of aerospace problems.[2] Steepest ascent algorithms have also been modified to accept branched trajectory problems.

Within the class of tug-assist-to-escape trajectories two cases will be considered. The cases differ in the constraints placed on spacecraft ignition and tug reignition points. In case 1, both points are left free. In case 2, the spacecraft is constrained to wait a fixed time in the intermediate orbit. In both cases the same equations of motion, initial and terminal boundary conditions, adjoint equations, performance index and terminal transversality apply.

The five variables required to specify the state are: r radius from Earth's center; v velocity; γ angle between velocity vector and local horizontal; ϕ range angle and m mass. The control θ is the angle between the thrust and the velocity vectors. T is thrust, I is specific impulse, μ the Earth's gravitational parameter and g_0 acceleration of gravity at the Earth's surface. Adjoint variables λ have a subscript indicating the associated state variable. The H-function is defined in the usual way: $H = LT/g_0 I + H_0$ where $L = g_0 I\lambda/m - \lambda_m$, $\lambda^2 = \lambda_v{}^2 + (\lambda_\gamma/v)^2$ and $H_0 = -\lambda_v \mu \sin\gamma/r^2 + \lambda_\gamma(v/r - \mu/r^2 v)\cos\gamma + \lambda_r v \sin\gamma + \lambda_\phi v \cos\gamma/r$. Initial and terminal times for the jth branch are a_j and b_j.

The initial state is completely specified. The final state of the tug including its mass is fixed except that total range $\phi(b_2)$ is free and time b_2 is also free. The final state of the spacecraft is constrained to meet some escape energy E. This formulation is range free and autonomous so λ_ϕ and H are identically zero.

In case 1, the intermediate orbit is a Kepler ellipse, thus coasting periods may be omitted by relating[2] the ends of the thrusting arcs and then deriving the jumps in the adjoint variables. The conditions relating states across a coast are simply statements of conservation of energy and angular momentum. The initial spacecraft mass is fixed at m_s so $m(a_3) = m_s$ and $m(a_2) = m(b_1) - m_s - \Delta m$ where Δm is mass lost during separation. Necessary conditions[2] applied to this case imply that $L(b_1) = 0$ which may be used to find the burn time of branch 1. Additional results assist in determining jumps in the costate across the coast; however, three jumps remain unknown.

For case 2 the previous formulation is modified to include a constraint on time the spacecraft spends in the intermediate orbit so the switching function for the termination of branch 1 becomes $H_0(a_3) - H_0(b_1) = 0$. Again necessary conditions provide information on the costate at the beginning of branches 2 and 3 but three quantities remain unknown.

To obtain numerical solutions the Newton method with some modifications was used. The branches are sequentially integrated in the order 1-2-3. Five initial values are unknown. Two unknowns may be related to $\theta(a_1)$ and $\dot{\theta}(a_1)$ while the remaining three may be taken as part of the costate at a_3. Branch 1 is terminated by a switching function, the second branch cuts off on mass and the third on energy. This leaves five unsatisfied boundary conditions, two transversality conditions at b_3 and three conditions constraining the tug to return to the base orbit.

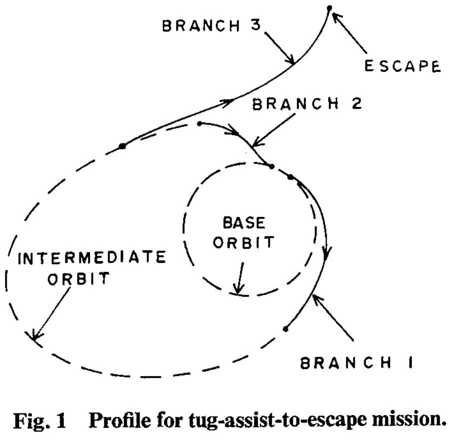

Fig. 1 Profile for tug-assist-to-escape mission.

Received September 27, 1971; synoptic received January 6, 1972. Full paper is available from National Technical Information Service, Springfield, Va. 22151, as N72-70369 at the standard price (available upon request).

Index categories: Lunar and Planetary Trajectories; Earth Orbital Trajectories.

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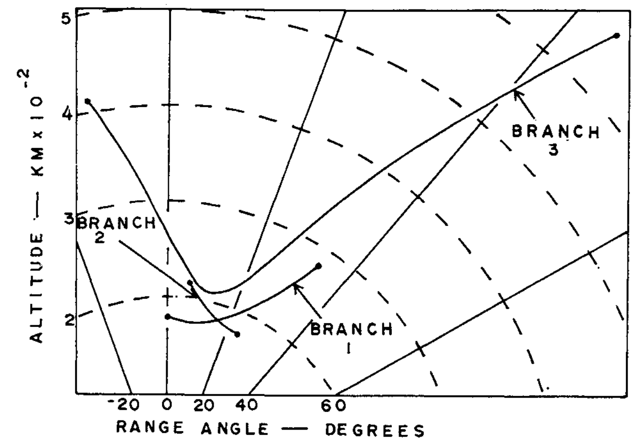

Fig. 2 Polar plot of altitude and range angle for typical case 1 solution showing three branches. Tug I_{sp} is 500 sec and spacecraft I_{sp} is 400 sec. Final energy is zero.

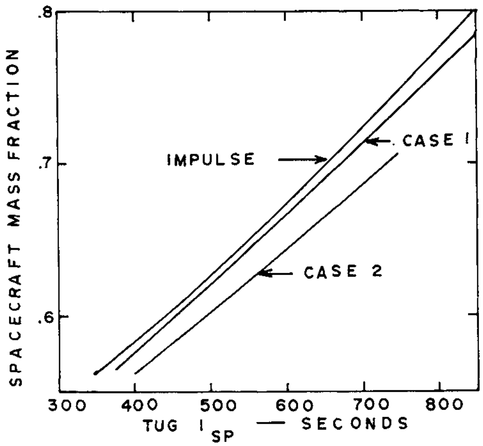

Fig. 3 Effect of tug I_{sp} on performance showing impulsive, case 1 and case 2 solutions. Spacecraft I_{sp} is held constant at 400 sec and final energy is zero.

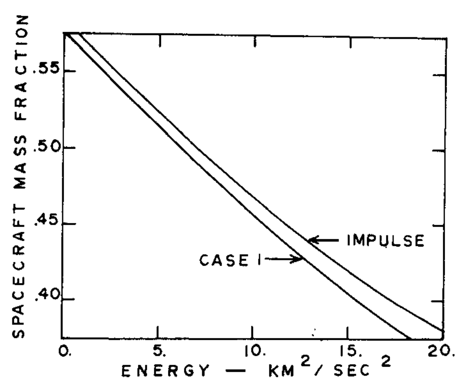

Fig. 4 Case 1 performance for varying escape energy compared with impulsive solution. Spacecraft and tug I_{sp} are both 400 sec.

A typical optimal trajectory for case 1 is given in Fig.2. The intermediate orbit is omitted for clarity leaving only the thrusting arcs. In all case 1 examples relight for tug and spacecraft ignition point occur before perigee. For the case shown the spacecraft ignites about 18° before the tug re-ignites so the tug may view spacecraft ignition. However, as tug I_{sp} is increased the two points interchange positions. Figure 3 compares finite burn results with impulsive performance as tug I_{sp} is varied. Figure 4 displays performance against escape energy. For these figures spacecraft mass fraction is defined as $m(b_3)/m(a_3)$ and the base orbit is a 100 naut. miles circular orbit. Spacecraft wet mass is taken as one quarter of tug wet mass and tug propellant ratio is 0.475. Initial thrust acceleration is 0.18156 g_0 and initial spacecraft thrust acceleration is 0.22695 g_0.

References

[1] Weyers, V. J. and Teren, F., "Optimal Round Trip Space Tug Trajectories for Earth Escape Missions," AAS Paper 71-370, AAS/AIAA Astrodynamics Specialists Conference, Fort Lauderdale, Fla., 1971.

[2] Vincent, T. L. and Mason, J. D., "Disconnected Optimal Trajectories," *Journal of Optimization Theory and Applications*, Vol. 3, No. 4, April 1969, pp. 263–281.